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On "Introducing Excess Return on Time-Scaled Contributions": *A Clarification*

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Jiang (2017) proposed an interesting performance metric for assessing the economic profitability of financial assets and portfolios, called *excess return on time-scaled contributions* (ERTC). This is a special case of a wider class of performance metrics called *average internal rate of return* (AIRR), introduced in Magni (2010). Special cases of AIRR only differ by the capital base, which may be chosen at the analyst's discretion: overall market value, book value, economic value, benchmark value, total or net contribution, initial contribution, etc. The choice of the capital base (and the use of discrete time or continuous time) is domain-specific: It depends on the field of application, the purpose of the analysis, the available data, the ability of making estimates of the asset's value, and so on.

In Example 11 (Jiang 2017, p. 86), the author deals with economic AIRR (EAIRR) and index-comparison-method (ICM)-based aggregate return on investment (AROI). The former is a special case of AIRR associated with economic values (see Magni 2013a, 2016), while the latter is a special case of AROI, itself an undiscounted variant of AIRR, which is associated with a benchmark portfolio (see Magni 2011, 2015; Altshuler and Magni 2015). The author warns the reader against potential violations of value additivity of EAIRR and ICM-based AROI. However, there is no potential violation of value additivity in either case, as we now illustrate, if the correct procedure for computing EAIRR and ICM-based AROI is followed.

EAIRR. Let 0 be the starting date of the analysis and n be the ending date of the analysis. For $t = 0$, the capital invested is equal to initial cash flow, f_0, changed in sign. The capital value at time $t \in \{1, 2, \ldots, n-1\}$ is the asset's *economic value*, defined as the discounted sum of prospective cash flows f_{t+1}, $f_{t+2}, \ldots, f_n$. The stream of beginning-of-period capital values is then $(c_0, c_1, \ldots, c_{n-1})$, such that

$$c_0 = -f_0, \quad c_t = \sum_{k=t+1}^{n} f_k\, d_{k,t} \quad \text{for } t \in \{1, 2, \ldots, n-1\}, \quad c_n = 0 \quad (1)$$

where $d_{k,t} = \prod_{j=t+1}^{k} (1+b_j)^{-1}$ is the value at time t of \$1 available at time k and b_j is the benchmark rate.[1]

ICM-based AROI. The capital value associated with the ICM-based AROI at time $t \in \{1, 2, \ldots, n-1\}$ is the replicating portfolio's value, obtained recursively as $c_t = c_{t-1}(1+b_t) - f_t$. The stream of ICM-based capital amounts $(c_0, c_1, \ldots, c_{n-1})$ may also be written as the accumulated cash flows from time 0 to time t:

[1] Lindblom and Sjögren (2009) called this capital stream *strict market depreciation*.

$$c_0 = -f_0, \quad c_t = -\sum_{k=0}^{t} f_k \; u_{k,t}$$

$$\text{for } t \in \{1, 2, \ldots, n-1\}, \quad c_n = 0 \tag{2}$$

where $u_{k,t} = \prod_{j=k+1}^{t} (1+b_j)$ is the accumulation factor from k to t.

The capital values presented in Jiang (2017, Exhibit 7) are not correctly computed according to the definition of EAIRR and ICM-based AROI. More precisely, assets A and B considered by the author have the following cash flow streams: $A = (-10, 0, 15, 0)$ and $B = (0, -10, 0, 8)$. Applying Equation 1 and using a 3% benchmark rate, as suggested by the author, the economic values are as follow:

Economic Assumption							
t	**A**	**B**	**A + B**	***c*(A)**	***c*(B)**	***c*(A) + *c*(B)**	***c*(A + B)**
0	–10.00	0.00	–10.00	10.00	0.00	10.00	10.00
1	0.00	–10.00	–10.00	14.56	**7.54**	22.10	22.10
2	15.00	0.00	15.00	0.00	7.77	7.77	7.77
3	0.00	8.00	8.00	0.00	0.00	0.00	0.00
			Aggregate capital:	24.14	14.64	38.78	38.78

The error lies in the computation of the second capital of asset B. Specifically, Exhibit 7 in the paper reports 10 instead of 7.54. The former is equal to the cash flow of B at the end of the first period, which is not the economic value. Noting that the prospective cash flows of B are $f_2 = 0$ and $f_3 = 8$, and using Equation 1, the economic value is $c_1(\text{B}) = \frac{0}{1.03} + \frac{8}{1.03^2} = 7.54$. Therefore, value additivity is preserved:

$$c_1(\text{A} + \text{B}) = \frac{15}{1.03} + \frac{8}{1.03^2} = 22.10$$
$$= 14.56 + 7.54 = c_1(\text{A}) + c_1(\text{B}).$$

(The aggregate capital is found by summing the present values of capital amounts.)

As for the ICM-based approach, the error is analogous and resides in the third capital of asset A, $c_2(\text{A})$. Using Equation 2, one must compute the accumulated sum (changed in sign) of the cash flows occurred from the starting date: $c_2(\text{A}) = 10(1.03)^2 - 0(1.03) - 15 = -4.39$. Therefore, value additivity is preserved:

$$c_2(\text{A} + \text{B}) = 10(1.03)^2 + 10(1.03) - 15 = -4.39 + 10.3$$
$$= 5.91 = c_2(\text{A}) + c_2(\text{B}).$$

ICM–Based Assumption							
t	**A**	**B**	**A + B**	***c*(A)**	***c*(B)**	***c*(A) + *c*(B)**	***c*(A + B)**
0	–10.00	0.00	–10.00	10.00	0.00	10.00	10.00
1	0.00	–10.00	–10.00	10.30	10.00	20.30	20.30
2	15.00	0.00	15.00	**–4.39**	10.30	5.91	5.91
3	0.00	8.00	8.00	0.00	0.00	0.00	0.00
			Aggregate capital:	15.91	20.30	36.21	36.21

(The aggregate capital is found by summing the capital amounts.) Exhibit 7 in Jiang (2017) reported 0 instead of −4.39.

As a result, the decomposition process proposed in Jiang (2017) is not necessary for value additivity of capital values: EAIRR and ICM-based AROI are additive (as well as any AIRR). (See also the treatment of portfolios in Magni 2013b.)

Further, we stress that any AIRR (and AROI) is consistent with known market values in an obvious way, for the AIRR (AROI) is an approach that allows for *any* capital value at *any* date. If market values are known only at some dates, it is natural to make use of them and decompose the investment into consecutive assets whose lives are delimited by the dates at which the market values are available (see Magni 2014, sec. 6). For each such asset, one may use any preferred AIRR; one can even mix different AIRRs and get a *blended* AIRR (e.g., Cuthbert and Magni 2016, 2018). In general, one may blend any AIRR, the choice being a matter of informed judgment and economic significance.

Finally, as shown in Magni (2016) and Magni et al. (2018), any rate of return is a special case of either AIRR or AROI.[2] Whether AIRR class or AROI class is more appropriate and which one among the various AIRR or AROI metrics should be selected is a matter of informed judgment (not a mathematical issue). Regardless of the choice, the performance metric will be value additive and NPV-consistent (and there will be infinitely many ways to build AIRRs or AROIs consistent with market values).

[2] In particular, Magni et al. (2018) showed that 11 well-known measures of economic efficiency used by practitioners and scholars (including simple Dietz and modified Dietz) are special cases of AIRR or AROI.

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